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Accurate Determination of the Volume of an Irregular Helium Balloon

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In a recent paper, Zable[1] described an experiment with a near-spherical balloon filled with impure helium. Measuring the temperature and the pressure inside and outside the balloon, the lift of the balloon, and the mass of the balloon materials, he described how to use the ideal gas laws and Archimedes' principal to compute the average molecular mass and density of the impure helium. This experiment required that the volume of the near-spherical balloon be determined by some approach, such as measuring the girth. The accuracy of the experiment was largely determined by the balloon volume, which had a reported uncertainty of about 4%.

We describe here a simple experiment to determine the volume of an irregularly shaped helium-filled balloon to an uncertainty of less than 2%. Our method also relies on the use of Archimedes' principle and the ideal gas laws. Its data analysis can be carried out at different levels, depending on the depth of the error analysis, and on the assumptions made in using the gas laws. For example, in the first approximation the ideal gas constant for dry air can be used, while in a more accurate calculation the gas constant for the specific conditions of relative humidity and ambient temperature can be determined.

The practical methodology of taking the uncertainty in each of the measurements and determining the uncertainty in the final calculations is a skill that is often overlooked in high school.

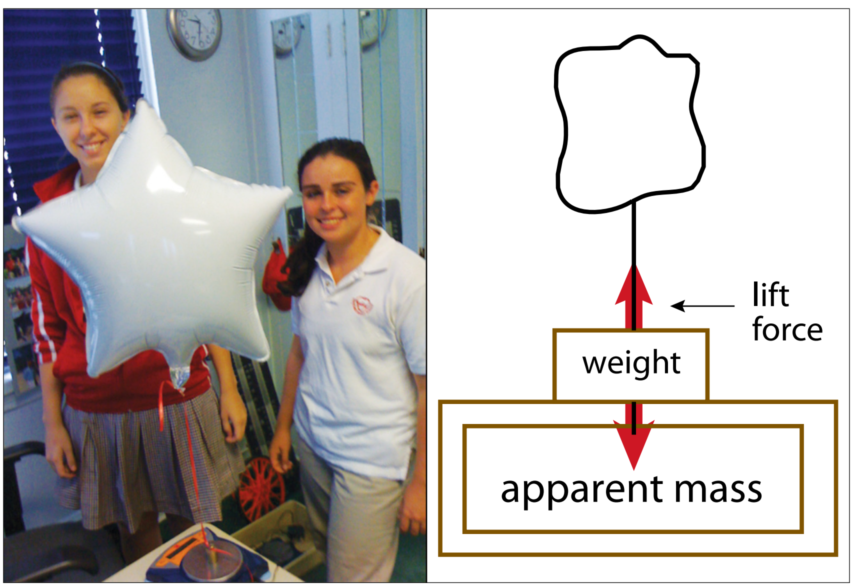

Fig. 1. Helium balloon and weight on balance.

The experiment

Our experiment needs the following items:

- an unfilled Mylar balloon,
- a simple mercury thermometer calibrated in degrees Celsius,
- an electronic balance that reads to 0.01 g,
- a small piece of ribbon or string, and
- a regulated tank of welding grade helium.

Since we did not have a barometer, we used the average of the reported atmospheric pressure at two weather stations that were close to the school and at nearly the same elevation. The setup is sketched and photographed in Fig. 1.

We measure the mass of a completely flat, empty balloon, the mass of a nominally 100-g mass, and the mass of the ribbon, which is used to connect the 100-g mass to the balloon. We next inflate the balloon with high purity helium to a little less than full, so that the pressure difference between the inside and the outside of the balloon will be small, here assumed negligible. In the Zable[1] paper, the pressure difference between the inside and the outside of a tightly stretched rubber balloon was estimated to be about 2.5%, so that in our partially filled, non-stretched Mylar balloon, the assumption of negligible pressure difference seems justified. One of the nice things about this experiment is that the balloon can take on any shape and we can still get an accurate calculation of the volume.

We attach the 100-g weight to the balloon with the ribbon as shown in Fig. 1, place the weight on the balance, and note the reading of the balance. The temperature of the air next to the balloon is also recorded. This completes the measurements needed to calculate the balloon volume. For a more accurate calculation of the volume, one needs to take into account the mole fraction of moisture in the air. For this, we either need to measure the relative humidity of the air or use the value reported at a nearby weather station.

Calculation of balloon volume

Let W_r, W_{bag}, W_{100}, and W_{scale} be the weights of the ribbon, the balloon material, the 100-g mass, and the balance reading when the inflated balloon is attached (converted to newtons by multiplying by g), respectively. In our experiments these were, in that order: 0.00265 N, 0.1202 N, 0.9823 N, and 0.9526 N.

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Denote by W_{He} and W_{buoy} the weight of the helium in the balloon and the buoyancy force, respectively. The latter is the weight of the surrounding air displaced by the balloon. We calculate these as follows:

Assume that air at ambient conditions will accurately follow an ideal gas equation of state:

$$P_{air}\, V = m_{air}\, R_{air}\, T, \tag{1}$$

where P_{air} is the atmospheric pressure in kPa, V the balloon volume, R_{air} the gas constant for dry air in kJ/kgK, and T is the temperature in kelvin. The dry air gas constant is 0.2870 kJ/kgK. The ambient temperature was 298.0 K, and the air pressure was 101.26 kPa (as estimated from nearby weather stations).

Inserting all the data, we get for W_{buoy}:

$$W_{buoy} = (11.62V)\ \mathrm{N}.$$

A similar application of the gas law for helium, with R_{He} = 2.077 kJ/kgK, gives:

$$W_{He} = (1.605V)\ \mathrm{N}.$$

The force balance in Fig. 1 is given by:

$$W_{scale} = W_r + W_{bag} + W_{100} + W_{He} - W_{buoy}. \tag{2}$$

Inserting all of the data into Eq. (2) and solving for V, we get the volume of the balloon as 0.0152 m^3 or 15.2 L.

Uncertainty analysis

Systematic error

In the above calculation of the balloon volume, we used the gas constant for dry air and ignored the effect of moisture in the air. This is probably fine for first-year physics or chemistry classes, since the systematic error is small (about 1.3% at 70% relative humidity and 298 K ambient temperature) and the classes probably haven't been introduced to the concept of ideal gas partial pressure and mole fraction. For more advanced classes, a correction to the ideal gas constant for dry air can be made as follows:

The gas constant R for a particular ideal gas or gas mixture is inversely proportional to the molecular mass of the gas. Thus, we must first find the molecular mass for moist air.

$$P_{H_2O} = P_T\, X_{H_2O}, \tag{3}$$

P_T = atmospheric pressure = 101.26 kPa
P_{H_2O} = partial pressure of water vapor at 298 K and 70% relative humidity = 2.22 kPa.
X_{H_2O} = mole fraction of water vapor in the air.

The partial pressure of moisture in the air is determined by multiplying the vapor pressure of water at 298 K (3.165 kPa) times the relative humidity (70%) to get 2.22 kPa. From the above equation, the mole fraction of moisture in the air is 0.0219. Thus the moist air is 2.19% moisture and 97.81% dry air. From this we can calculate that the average molecular mass of the moist air is 28.73 g/mole when the molecular mass of dry air is 28.97. This results in the moist air gas constant being 0.2894 kj/kgK rather than the dry air value of 0.2870. If we use all of the measured values for temperature, pressure, etc. and include the R value for moist air, then the calculated volume of the balloon is 15.4 L rather than 15.2 L, which was obtained assuming dry air. Thus using the dry air gas constant under these conditions introduces a systematic error of about 1.3%.

Random error

The uncertainty in the measured parameters of this experiment are assumed as follows:

T = ±0.5 °K
P = ±0.1 kPa (not measured on site)
W_{bag} = ±0.0001 N
W_{ribbon} = ±0.0001 N
W_{100} = ±0.0001 N
W_{scale} = ±0.0005 N (reading jumps)

If the actual temperature is 0.5 K higher than measured, then the actual balloon volume will be higher than the calculated volume. Similarly if the actual mass values of the bag, ribbon, and weight are greater than measured, then the actual balloon volume will be higher than calculated. For pressure, if the actual pressure is 0.1 kPa lower than measured, then the actual volume will be higher than calculated.

The uncertainty in the W_{scale} measurement was much larger than the uncertainty in the other mass measurements because when the balloon is attached to the weight sitting on the balance, the air currents cause the readings to jump around. A smaller value of W_{scale} leads to a larger balloon volume.

When we apply the maximum uncertainty to all of the measurements that will result in a bigger balloon volume and also use the value of R for moist air, then we get the biggest possible balloon volume when plugging these values into the above equations. This results in a maximum balloon volume of 15.5 L as compared to the original calculated value of 15.2 L. Thus, the maximum volume uncertainty in the example experiment and calculation, using the R value for dry air, is about 1.9% (15.2 ± 0.3 L). If we make the correction for moisture in the air, then the maximum uncertainty is about 0.7% (15.4 ± 0.1 L).

Conclusions

The experiment and calculations described herein would be valuable to beginning experimental science classes, excluding the section pertaining to uncertainty analysis and the calculation of the gas constant for moist air. It provides a simple, interactive way for students to learn about Archimedes' principle and the ideal gas laws. More advanced classes can perform the detailed uncertainty analysis and make the corrections to the gas constant for moisture in the air. At either level, this technique provides a way to determine the volume of an irregular shape with an uncertainty of less than 2%.

Instructor's Note: The experiment described herein and the accompanying calculations and error analysis were developed by two seniors and their instructor at Mayfield Senior School in Pasadena, CA, as part of a special study program introducing engineering thermodynamics.

Reference

1. Anthony C. Zable, "Experiments with helium-filled balloons," *Phys. Teach*. **48**, 582–586 (Dec. 2010).

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